PROSPECTIVA PINGENDI, PROSPECTIVA FINGENDI
FOR A HISTORY OF THE DIFFERENT RULES OF PRACTICAL PERSPECTIVE

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HISTORY OF PERSPECTIVE
LEON BATTISTA ALBERTI
THE TWO RULES OF PRACTICAL PERSPECTIVE
OF JACOPO BAROZZI DA VIGNOLA
EGNAZIO DANTI
AMBROGIO LORENZETTI

The method of perspective representation has consolidated over time, up to its complete formalization, through a series of important and complementary achievements both in the scientific and artistic fields. While the study of the laws of ancient optics, combined with practical experimentation, slowly contributed to the rigorous formalization of the method in Renaissance intellectual circles, the workshop practice required operational rules that quickly and easily enabled artists to produce images in which the depth of the space and the three-dimensionality of the subjects it contains were evident.

The study presented here intends to focus attention on the procedures practiced by artists, with particular attention to perspectives rules that really weren't , in the scientific sense of the term, but which contributed to establish a shared and widespread basis for the development of sensitivity of seeing and representing in perspective, in the intimate and labile boundary between *prospectiva pingendi* and *fingendi*.

INTRODUCTION TO THE STUDY

The history of perspective has ancient roots, but only in the Renaissance, with illustrious scholars such as Filippo Brunelleschi, Leon Battista Alberti and Piero della Francesca, it begins to find a significant formalization –textual and graphic– in the context of scientific literature, together with the progressive possibility of a widespread diffusion, from the greatest humanist scholars, to workshop artists. As Piero della Francesca points out in the introduction to the third book of *De prospectiva pingendi* (Gizzi et al., 2016), for a correct practice of this art it is necessary to know the laws of perspective, "*senza de la quale non se po alcuna cosa degradare giustamente*" [without which nothing can be rightly degraded]: "*dico essere necessaria la prospectiva, la quale discerne tucte le quantità proportionalmente commo vera scientia, dimostrando il degradare et acrescere de onni quantità per forza de linee*" [I say that perspective is necessary, which discerns all quantities proportionally as a true science, demonstrating the degradation and increase of each quantity by force of lines] (p.153). Piero then continues with a harsh criticism of less rigorous artists, who appear unjustly praiseworthy in the eyes of those "*che non ano notitia de la virtù de l'arte con falso giuditio*" [who do not know the virtue of art, with false judgment] (p. 153).

A century later, we find similar concepts expressed in Danti's comments on Jacopo Barozzi da Vignola's *Two Rules of Practical Perspective* (Barozzi, 1583/1974), in which the mathematician praises Vignola's work for defining two rules "*elette per ottime*" [elected for excellent] (p. 52), on which other procedures depend, with the exception of those that are defined as "false" (Barozzi, 1583/1974, pp. 84, 85), although still very widespread and practiced at the time.

However, carefully considering and observing the evolution of methods for the representation of space in ancient and medieval proto-perspectives (think for example of Pompeian architectural perspectives, or the more recent works by Giotto, Duccio di Buoninsegna, Ambrogio Lorenzetti, to

name some of the most famous painters) we can noted that the history of perspective, in the broadest and most primordial sense of the term, also passes through the empirical procedures. These procedures contributed to the development of the primal intention inherent in the act of subjective representation of space, dependent on the position and personal perception of the artist who paints it, which can then be replicated in the observer to whom the artwork is aimed.

The progressive formulation of the geometric principles that contribute to the definition of the actual method and the simultaneous existence of these different practical procedures show a subtle but fundamental separation between science and art of drawing. The scientific evolution of perspective thought had led in just over a hundred years to the definition of rigorous procedures such as Alberti's legitimate construction, the two "ways" of Piero della Francesca and the *Two rules* of Vignola. At the same time, the needs of artistic practice had favoured the invention and diffusion of alternative ways of perspective representation, some of which were decidedly approximate, although effective.

This dichotomy is also evident in the treatises, in which, starting from the sixteenth century, we note the development of distinct strands or independent sections within the individual texts, in order to embrace a heterogeneous audience. In fact, not all the readers were evidently in a position to understand or want to apply themselves in the deepening of the scientific principles of perspective representation, and, as the very structure of Vignola and Danti's *Two Rules* demonstrates, the texts were composed by providing different levels of depth. This made it possible to meet both the needs of those who claimed to understand the geometric principles underlying the procedures, and those who needed to use, in workshops, practical rules for the realization of the commissioned works, in a quick, simple and easily replicable and transmissible way. In fact, in the face of a limited implementation effort, thanks to more or less legitimate simplified procedures, it was possible to obtain perceptually very effec-

tive perspective representations, even if not perfect from a purely geometric-projective point of view, to use a term that will construct the apex of the scientific development of the method in the nineteenth century.

APPROXIMATE PROCEDURES

Severely rejected by the intellectuals of the time, who aimed at the transmission of scientific principles or at most at the dissemination of rules that would make their application easier but always conform to rigorous constructions, today the rules defined as "false" are not less interesting. Like the right ones, they can also contribute to understand the evolution of the method of representation within which they were born. A *prospectiva fingendi* that is a prelude to *prospectiva pingendi*.

The false rules, in spite of themselves, had the merit of constituting a fundamental basis, a crucial transition moment, for the first pictorial works that contributed to the diffusion of perspective sensitivity. For example, let's focus on the period immediately preceding the consecration of the method, a period in which, even with an awareness that is still incomplete, it is evident the intention to represent three-dimensionally the space in which the portrayed figures are immersed, as happens in Giotto, Duccio or Lorenzetti's work.

Let us consider a famous emblematic example, which often recurs in critical literature (Panofsky, 1927; Damish, 1995; Kemp, 1994/2005; Andersen, 2007): the *Annunciation* by Ambrogio Lorenzetti painted in 1344 and kept in the National Picture Gallery of Siena. As in other similar and coeval works, also in this case at least a couple of achievements are evident from the point of view of perceptual awareness of space. The plausibly square tile flooring, arranged parallel to the painting, reveals on the one hand the convergence of perpendicular lines in a point that is in the center of the scene, facing the observer, on the other hand that there is a progressive degradation of the intervals between the lines parallel to

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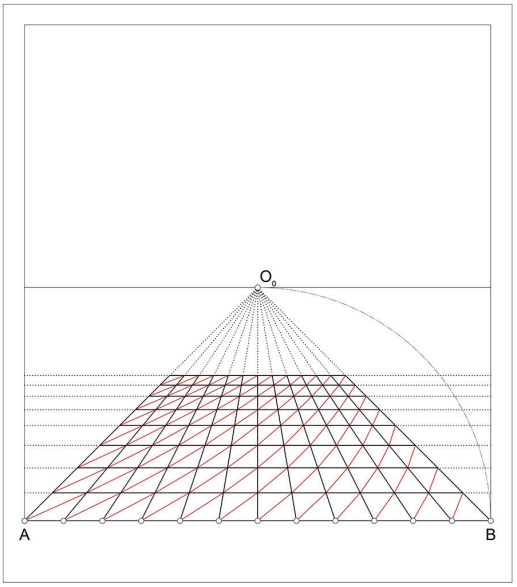
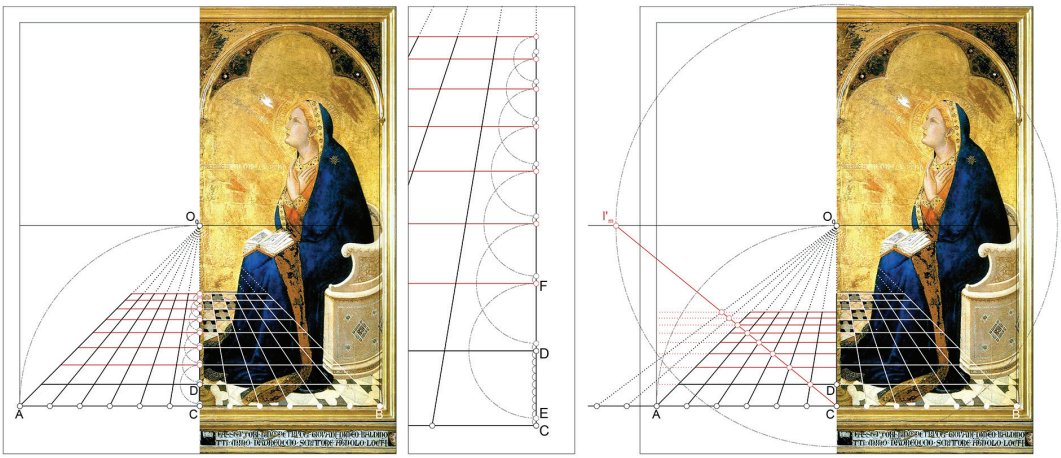


1b

Figure 1 Essential elements of the perspective representation of the floor in the *Annunciation* by Ambrogio Lorenzetti, 1344, National Picture Gallery of Siena. Author's elaboration.

the painting, giving a feeling of depth (Figure 1a). On closer observation, however, we understand that this depth is not calculated on the basis of a correct perspective degradation law: we can easily experiment this assertion by verifying the lack of convergence in a point on the horizon of the diagonal lines of the individual tiles (Figure 1b). Rather than noticing the similarities with a rigorously constructed perspective, it is however interesting, as we foregoing, to go in search of the possible rule –evidently empirical, but effective in its own way– used to determine the degradation of the intervals. With a scrupulous analysis, we may suppose the possible algorithm for generating the geometric structure (Figure 2a). We start from the arbitrary determination of the width of the painting and from the location of the principal point on the axis of vertical symmetry, at a height equal to half of the ground line (which is not unusual, as we will see, in the practical procedures described in perspective treaties). The ground line is divided into 12 equal parts, corresponding to the width of the floor tiles. The lines perpendicular to the picture, on



2a 2b

Figure 2 Deduction of the rule used by Lorenzetti for the degradation of depths and comparison with the construction of the same subject carried out in a rigorous way. Author's elaboration.

which the rows of tiles are arranged, are easily identifiable by tracing straight lines from these points to the principal point. We now come to the determination of the lines parallel to the painting, equidistant in reality, but not in perspective. The first CD interval, which defines the depth of the first row of tiles, is set arbitrarily. The subsequent ones are progressively and constantly decreased, compared to the interval that precedes every one of them, by the tenth part (CE) of the first interval. It is evident that one of the main limits that can be recognized in this rule consists in the fact that, since the decrease is constant, we reach a limit condition –well before approaching the horizon– in which the distance between two successive parallel lines is less than decrease to be applied. Wanting to compare the construction with the result that would be obtained by proceeding with scientific rigor, placing as a condition the equality of the first interval CD (Figure 2b), we note in fact that the prospectical acceleration should be greater, leading to the representation of a floor that it appears more shortened, less deep.

Procedures like this, which can be experimentally deduced *a posteriori* by analysing the works, have found space –albeit limited and usually with a critical attitude– in treatises on perspective, starting with Leon Battista Alberti (Al-

berti, 1435/2011). In paragraph 19 of the first book of *De Pictura*, he describes an erroneous procedure for the determination of the perspective intervals, based on subsequent subdivisions he calls "*superbipartienti*" [super-biparticular] (integer plus two thirds). It is therefore interesting to go in search of these procedures, which have a lot to tell about *perspectiva fingendi* and which can constitute an important basis for comparison and verification with respect to pictorial production characterized by only partially rigorous constructions.

This study intends to focus on three procedures in particular that are present in two fundamental testimonies of the history of perspective: the first in Leon Battista Alberti's *De Pictura*, the others in the *Two rules* of practical perspective by Jacopo Barozzi da Vignola and Egnazio Danti. These "false rules" (Barozzi 1583/1974, pp. 84, 85) –purely pragmatic and approximate with respect to scientific principles, but important for the wide diffusion that they had at the time, according to the authors– will be graphically analysed and consider in comparison to the rigorous construction, reserving also some surprises, as we will see in the case of the last rule.

ALBERTI'S APPROXIMATE PROCEDURE IN *DE PICTURA*

De Pictura, a work that Alberti wrote in the first half of the fifteenth century dedicating it to his friend Filippo Brunelleschi, opens with a first book that deals with the description of the legitimate construction underlying the perspective representation. Alberti, as already mentioned, also highlights a practical procedure whose result does not conform to this construction. The procedure (Figure 3) is applied to the construction of a grid with a square mesh placed on the ground plane, in frontal position respect to the picture plane. Arbitrarily traced the ground line and the horizon in relation to the height of the observer, and placed the principal point in a central position on the horizon, we represent equidis-

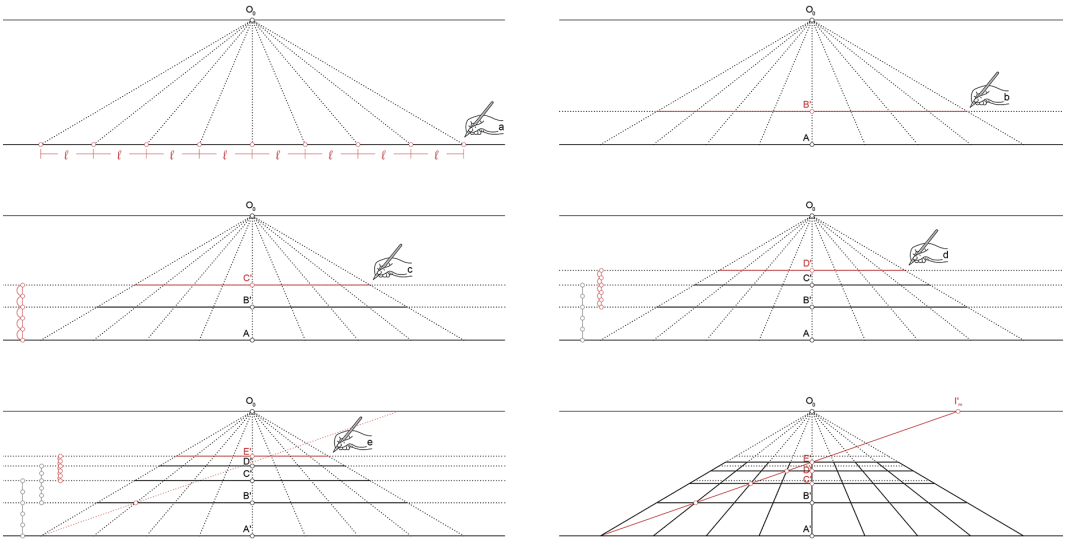


Figure 3 Illustration of an approximate procedure used in Alberti's time (*De Pictura*, paragraph 19).
Author's elaboration.

tant points on the fundamental line, depending on the width of the squares to be drawn. After tracing the perspective of the lines perpendicular to the picture passing through the aforementioned points and converging in the principal point, we move on to the determination of the apparent depths. The depth of the first interval is defined arbitrarily, while the subsequent ones are progressively reduced by one third with respect to the immediately preceding depth. As with Lorenzetti's *Annunciation*, here too the approximation of the construction is evident when we draw the 45° diagonals. However, unlike the case analysed above, the type of red uction applied theoretically allows in this case to progress to infinity with the definition of the intervals in depth, since each interval decreases proportionally with respect to the previous one, thus tending to the horizon. Finally, if we build the rigorous perspective of the same floor, starting from the same interval A'B', we note once again that its representation appears more foreshortened and the floor is less deep.

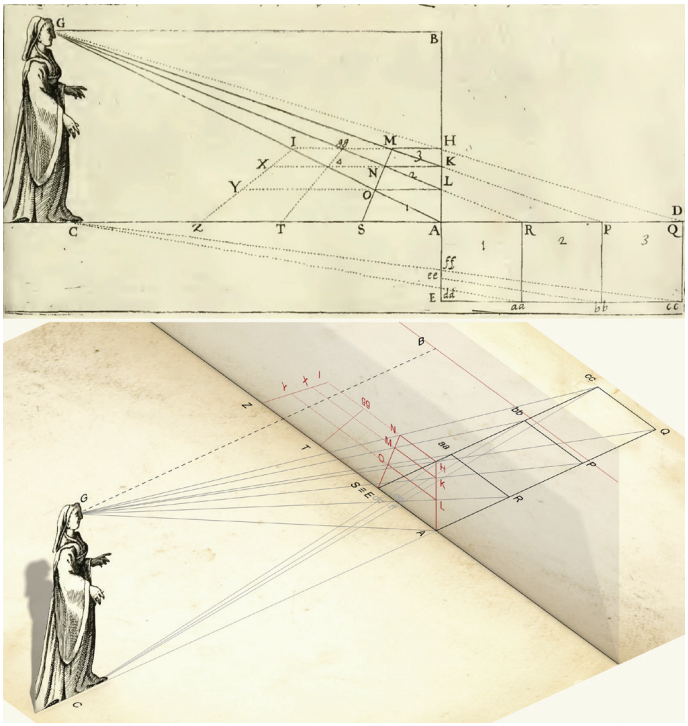


Figure 4 Illustration of the 'first rule' by Jacopo Barozzi da Vignola. Author's elaboration.

THE TWO FALSE RULES IN THE TREATY OF VIGNOLA AND DANTI

The treatise *Two rules of practical perspective* by Jacopo Barozzi da Vignola, published posthumously in 1583 by the mathematician Egnazio Danti, represents a valuable source of information on the evolution of perspective. In the text, art and science of drawing reflect each other through the mirror of perspective thinking, representing with great systematicity and clarity the state of technical and scientific knowledge consolidated up to that moment.

The structure of the treatise is indicative of the flexibility of consultation that is reserved for the reader, according to his needs, as Danti explains (Barozzi, 1583/1974, preface). The first part consists of a substantial theoretical introduction consisting of definitions, theorems and problems aimed

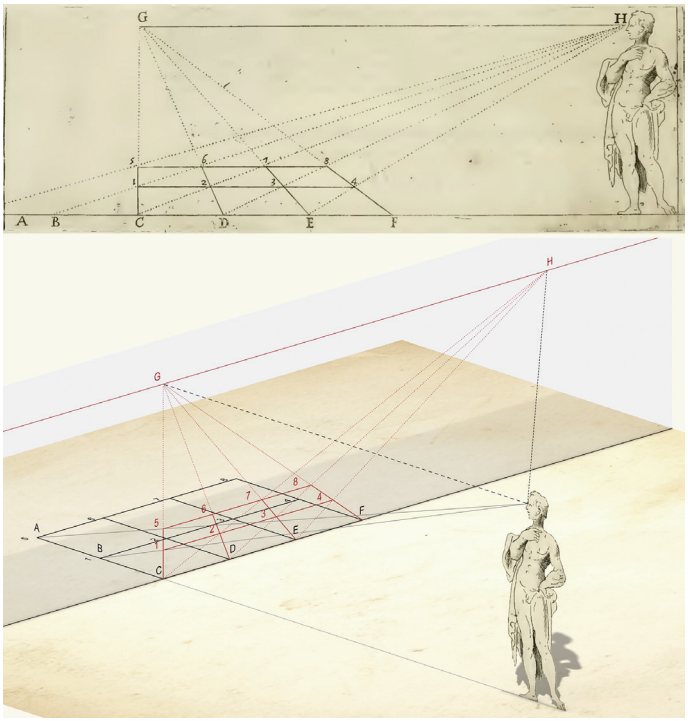


Figure 5 Illustration of the 'second rule' by Jacopo Barozzi da Vignola. Author's elaboration.

in particular at those who wish to understand the geometric nature of Vignola's rules. The second part, dedicated to who needs to learn only the practical aspects of the art of perspective, is instead made up of the illustration of the *Two rules*. Furthermore, for the artists *"che più si dilettano di operare, che di fare studio in diverse regole"* [who are more delighted to operate than to study the different rules], Danti thinks of a fruition of the text that exclusively contemplates the part relating to the second rule, which he considers *"più eccellente, & più facile di qualunche altra regola; con la quale potranno perfettamente operare, & ridurre qual si voglia cosa in Prospettiva"* [more excellent, and easier than any other rule; with which they will be able to work perfectly, and represent everything in perspective] (Barozzi, 1583/1974, preface).

As the title itself suggests, the treatise is known in particular for the formulation of two rules. The first, simple but

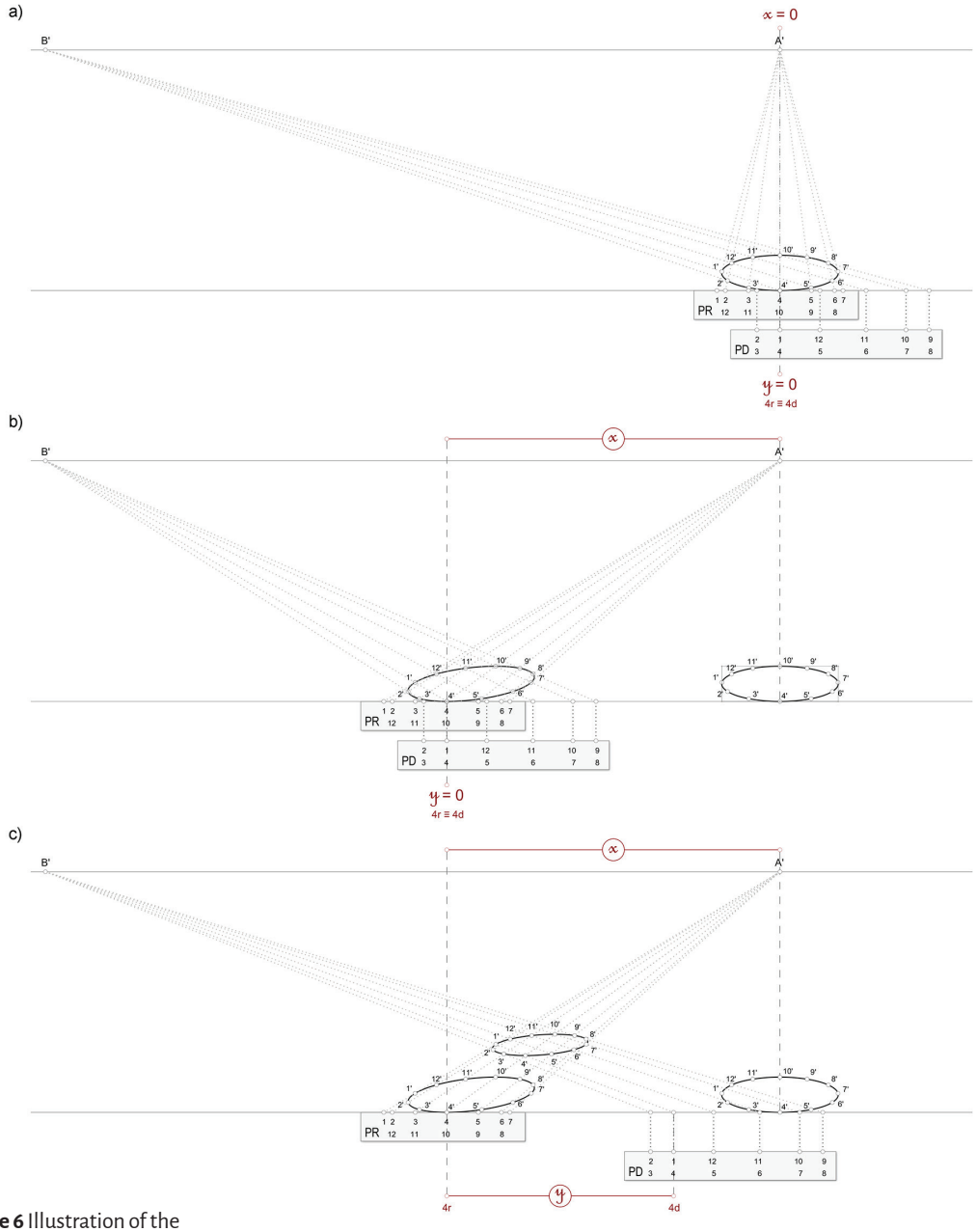


Figure 6 Illustration of the process of construction of the perspective with "*sagme*" as described by Jacopo Barozzi da Vignola. Author's elaboration.

laborious, uses projecting lines in plan and elevation to determine the position of the points in the perspective space (Figure 4). The second, more scientifically complex but more immediate, explicitly illustrates for the first time the construction of the perspective with the complementary help of the principal point and the distance point, intended, in this case, as the point where concur the straight lines inclined at 45° with respect to the picture plane (Figure 5). As part of this second rule, the authors describe a practical procedure, little known but noteworthy, linked to the use of "*sagme*", lines of paper on which the data for the discrete representation of a given subject are recorded, regardless of its position with respect to the observer and the picture plane, and therefore valid to represent innumerable perspectives of that subject by establishing *a posteriori* its position with respect to the observer and the picture (Figure 6) (Romor, 2019).

Deepening the critical reading of the treatise, we learn that these procedures are only two of the "different rules" –although all the others depend on them– with which we can make perspective drawings. In fact, in addition to these two, considered "excellent", Danti and Vignola provide examples of other rules, called "ordinary", illustrated in the mathematician's comments.

In the concluding comments on the first rule, Danti introduces two rules that he defines as false (Barozzi, 1583/1974, p.84, 85), illustrating them and explaining the reasons for non-compliance with the ordinary rule.

The first is described as "*tenuta in gran conto*" [highly regarded] by artists, but misleading for "*chi brama di ben operare*" [who desires to do well] (Barozzi, 1583/1974, p.84). Let's briefly analyse the steps of the procedure (fig. 7).

1. We consider B as the principal point and construct the objective sides of the squares on the ground line AC; even if it is not specified, from the following construction it can be deduced that AC must be equal to AB.
2. We trace the lines from E, F, G, H, I, C up to B.
3. We draw a quarter of a circle pointing at A with radius AB (= AC) from B to C and divide it into 15 parts.

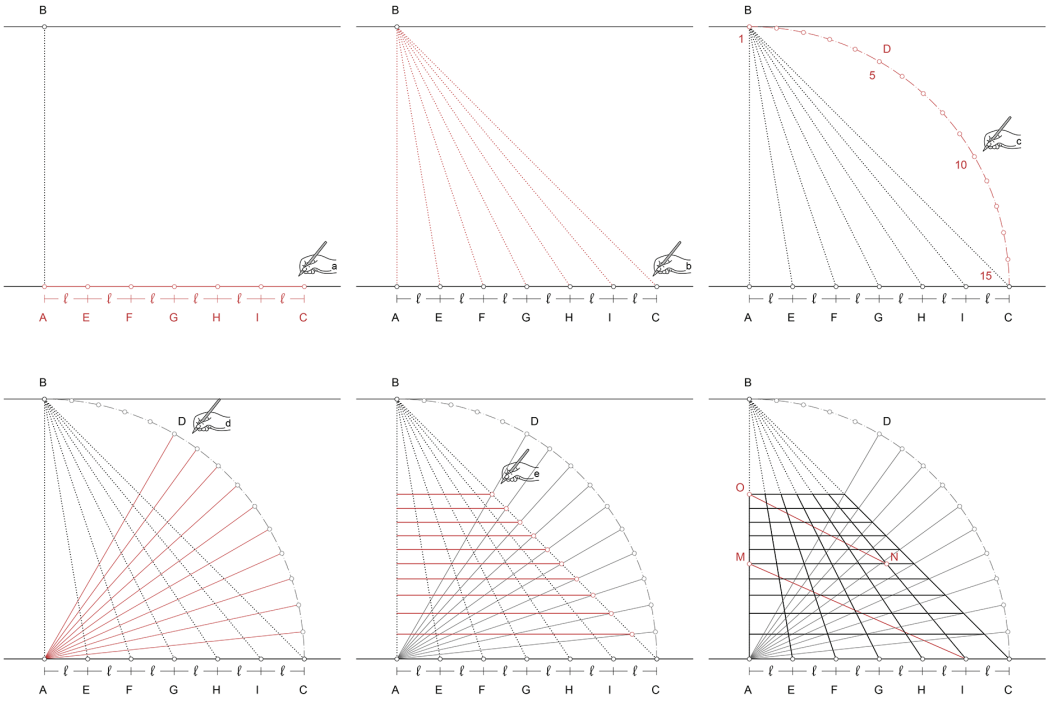


Figure 7 Illustration of the perspective construction with the first false rule described by Danti. Author's elaboration.

4. We place point D on the arc, which is one third (or "*anche una particella in meno*" [even one less particle]) of the arc length starting from B. From point D we draw a straight line towards A, and so for all the other points from D to C.
5. Where these lines intersect BC line, we conduct lines parallel to AC, which define the heights of the squares in perspective. The heights of the squares depend on the amount of points at which you decide to divide the BDC quarter circle.

Danti notes that this procedure cannot be consistent with the principle of degradation expressed in the introductory theoretical part and does not operate in accordance with the other rules. Furthermore, as proof of the falsity of the rule, he invites the reader to an experimental verification:

- We draw the diagonal (which is therefore directed towards the distance point).

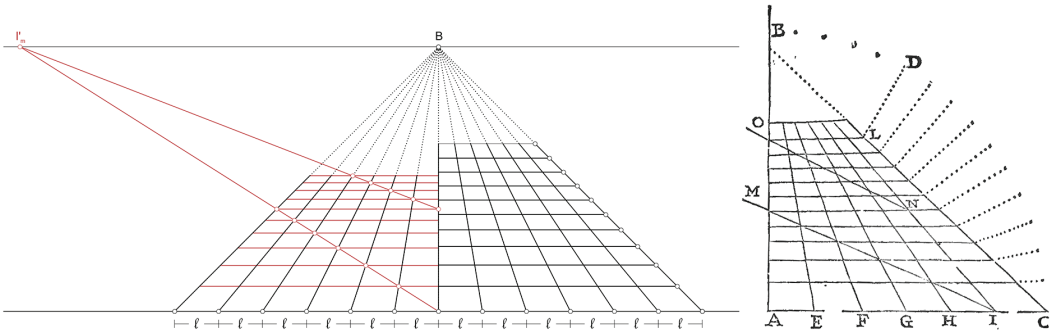


Figure 8 Comparison with the perspective construction of the same subject of figure 7 conducted in a rigorous way. Author's elaboration.

- From N (aligned with M) we draw a line towards O (five squares above M): this line does not pass through the diagonal edges of the squares and does not arrive at the same point where IM meets the horizon line.

Let's compare the result obtained instead with a rigorous construction, always starting from a first common interval (Figure 8). Again, the floor would be more shortened.

A second false rule is then introduced, which applies in the case in which squares of different sizes are to be represented. Danti emphasizes that this rule is also widely used by artists, from whom he himself learned it as correct, only to realize, with experience, its incompatibility with scientific principles (Barozzi, 1583/1974, p. 84, "*molto usata dagli artefici da' quali io già l'imparai per buona, e poi m'avvedi della falsità*" [widely used by artists from whom I learned it for good, and then I realized the falsity]). Danti than continues to underline that "*Non dobbiamo dunque meravigliarci, se bene spesso vediamo delle Prospettive inette, e malfatte, poi che si trovano de gl'artefici, che usono regole così triste*" [We should therefore not be surprised if very often we see inept and bad perspectives, since there are authors who use such sad rules].

Let's analyze the procedure step by step (Figure 9).

1. We establish the main point C. We draw the ground line RB. We draw line CA (perpendicular to RB).
2. We choose point D on CA, such that CD is a third of CA. We trace BC and BD.

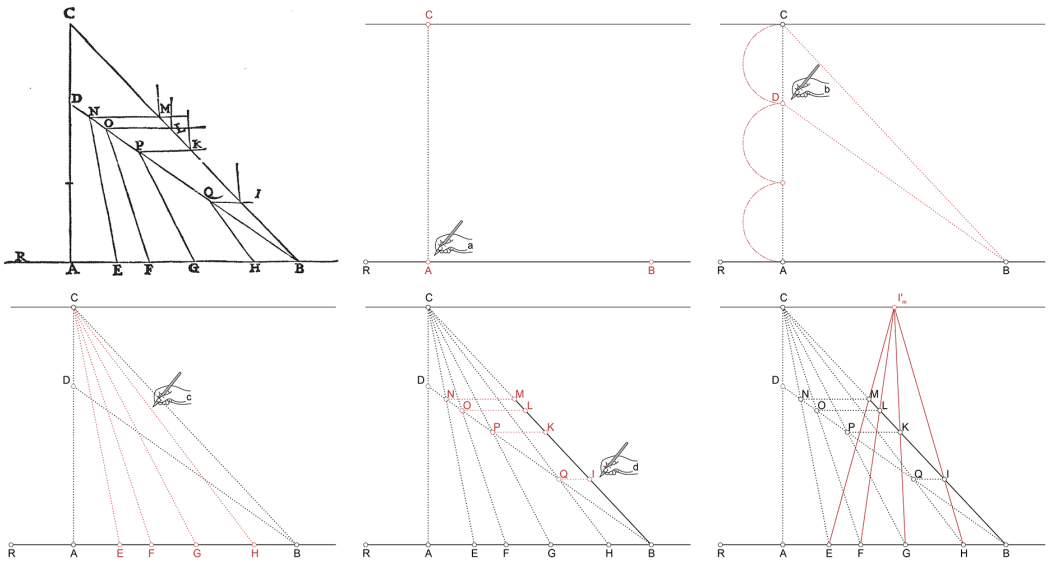


Figure 9 Illustration of the perspective construction with the second false rule described by Danti. Author's elaboration.

3. We report on RB the sizes of the squares (or buildings, as Danti specifies) which must then be reported on BC (points E, F, G, H). We draw the lines from E, F, G, H up to C, which intersect the BD at points N, O, P, Q.
4. From N, O, P, Q we draw lines parallel to AB, which intersect the BC at points M, L, K, I, which provide the measurements of the depths to be represented in perspective.

As Danti correctly observes, the depth of the foreshortening depends on where point D is placed, higher or lower than the principal point. Proof of the falsity of the rule, according to Danti, would lie in the fact that the measures are not proportionally degraded because IB>HB (HB is the true measure of IB, so he says that this thing is absurd) as IK>HG, while KL<GF and LM<FE. We have used the conditional because in reality –and here lies the surprising fact anticipated in the introduction to the study– this procedure turns out to conform to the laws of perspective, to an in-depth analysis: Danti sees a falsehood in a rule that he had used for years ignoring having worked right. The confusion is probably justified by the choice of the perspective foreshortening in the example illustrated,

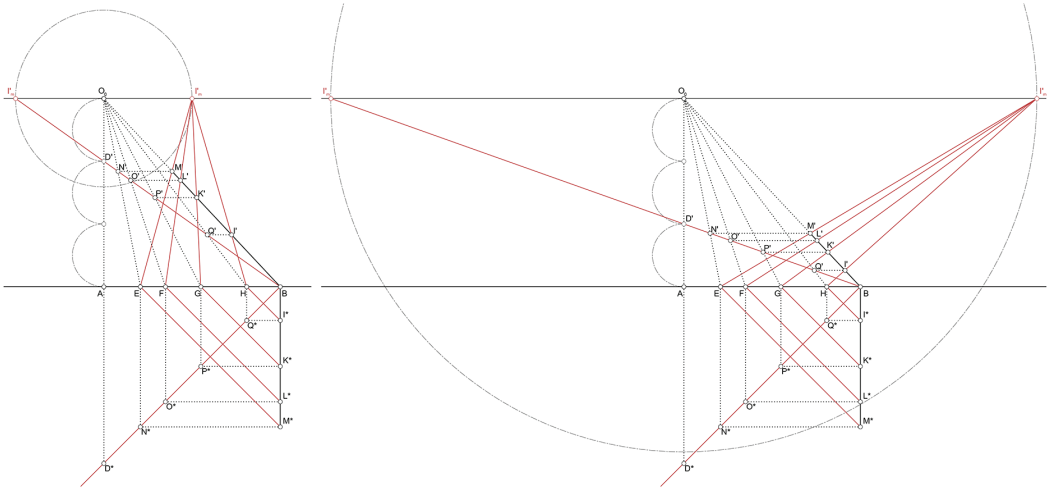


Figure 10 Comparison with the perspective construction of the same subject in figure 9 conducted in a rigorous way. Author's elaboration.

which actually generates the disproportion between the perspective segments described by Danti. Let's deepen why the procedure is correct. First of all, by reconstructing the lines joining the extremes of the measured segments (for example L'M') and their projections on the picture (for example EF), we observe that these lines converge at a point on the horizon (Figure 10). Today, we define this point the measuring point of perpendicular lines, corresponding to the distance point in Vignola's second rule. We then note that also the straight line BD' is diagonal of the squares traced in the perspective (for example BEN'M') and that therefore it will intersect the horizon in the other distance point opposite to the first. Therefore, by tracing the distance circle, having as radius the interval between this point and the principal point, we realize that the measured segments fall well outside the circle, thus giving rise to the deformations mentioned by Danti, which however are only apparent: the procedure acts in accordance with the perspective principles. In fact, if we choose, for example, to place the point D at the extreme of the first third between A and Oo, consequently, the main distance significantly increases and the apparent deformations detected by Danti disappear, being the perspectives of the measured segments all shorter than their projections on the ground line.

Given the correctness of the rule, we must investigate the question of the location of point D, on which the perspective view depends, as Danti says, and therefore the principal distance too. Thanks to the principle of similarity between triangles, which in the Renaissance was used to explain the laws of perspective (for example in Piero della Francesca and Danti himself), it is in fact possible to control the principal distance during the design phase by establishing the height of point D'. Let's consider the triangles D'AB and D'OoI'm: the OoI'm segment is proportionate to the AB segment as is the D'Oo segment with respect to D'A. For example, in the case illustrated by Danti, the principal distance measures half of the segment AB, while in the second hypothesis it is double compared to it.

At this point, it appears evident how the empirical but rigorous procedure allows overcoming a recurring problem in the construction of perspectives, especially architectural ones on 1:1 scale: the inaccessibility of vanishing points. The measurement process described, in fact, taking advantage of the control of the foreshortening thanks to the similarity highlighted, does not require the distance points to be materially present on the support to be painted.

CONCLUSIONS

The study presented here aims to focus on 'ordinary' procedures, more or less rigorous, which are considered in the treaty alongside the two main rules.

If on the one hand it is interesting to understand the logic and application of approximate procedures, the story of the tradition of Danti's alleged false rule, which deceives him in evaluating its actual correctness, is evidently significant of how labile it was in the sixteenth century, in perspective, the boundary between *fingendi* and *pingendi*. A border that today we can well delineate with the awareness of the achievements of projective geometry and the development

of the method of direct perspective, but which at the time was characterized by the mutual influence of the evolution of the principles of ancient optics and of the artistic workshop experiments. These rules lead to the creation of representations that are not at all perspectives from a purely scientific point of view, but which become so to the extent that they are able to generate an image that perceptually refers to a perspective space. Rules that, moving from the *prospectiva pingendi* to the *prospectiva fingendi*, acquire considerable importance, given the versatility, immediacy and constructive simplicity that have guaranteed and favoured a wide diffusion, placing them in the history of perspective thought in the same way as nobler processes.

This study hopes, in its future development, the construction, over time, of an abacus of prospective procedures, whether rigorous or approximate, born within the needs of the workshop practice, which can be a useful tool for comparison and verification with respect to coeval pictorial production.

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