HOMOLOGICAL RELATIONS BETWEEN GRAPHIC-GEOMETRIC DESCRIPTIVE / PROJECTIVE MODELS

PRINCIPLE OF VIRTUAL WORKS IN DESCRIPTIVE GEOMETRY

Andrea Donelli
University of Trento
Department of Civil, Environmental and Mechanical Engineering
andrea.donelli@unitn.it

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DESCRIPTIVE GEOMETRY
GRAPHIC-GEOMETRIC MODELING
REALITY
VIRTUALITY
HOMOLOGICAL RELATIONS

The concept of the model is vast and complex and in its scientific and empirical implications and applications or on an experimental basis refers to various disciplines and knowledge. It essentially concerns questions relating to how to elaborate conjectures, thoughts, observations and geometric graphic operations as well as research spaces where there are considerations that involve and that take as the core of the theme the investigation that directly concerns the concept and the actual implementation of the model. The question relating to the concept of model that can evoke interest and stimulus for further study consists in the fact that thinking about the model in descriptive geometry contains two aspects connected to each other. The first concerns the structure of the representation system, that is the ordering relative to the representation system on which the method is arranged graphically and geometrically; the second concerns the drawn visualization referred to the descriptive procedure of the represented image and attributed to the outlined object. Descriptive geometry virtually creates these elements using the founding principle of projection and section. In fact, modelling through descriptive geometry allows, in its different forms, for the representation of geometric figures constructed through the operations of projection and congruence. This is, therefore, a concept that determines an inseparable unity that allows one to virtually establish the geometric graphic problem parallel to the reality.

INTRODUCTION

In Descriptive Geometry the rules, laws, and principles that constitute both the foundations of the discipline and its application according to methods and procedures prescribe, in any exercise or demonstration concerning the graphic-geometric representation, the relation of the images, figures or the drawing itself, referring to architecture, through the conjunction of homologies which are completed, thanks to the support of one or more systems connected to the descriptive and projective geometric construction. In this way, the images are subjected to a modelling process whose derivation is the result of precise and foundational rules that are fixed on an order, or on codes and methods that are applied through very specific procedures. In this affirmation, it is possible to include comparisons in the graphic-geometric study; above all, results can be obtained by means of appropriate superimpositions passing from one representation to another for the effects produced by each single exercise corresponding to a method and a specific procedure, which in the same way is also contiguous in helping to determine the very effects of the representation. In this way it is possible, through the geometric articulation obtained through to morphological modelling, to match both the structure of the representation system and the different morphologies which represent the different passages relating to the image or to the different graphic descriptive phases that are intended to be disclosed, since for introspective reasons they are considered to be inherent to the research or specific investigation of a study path related to the architecture. In this aspect a further consideration emerges due to the question of virtuality, since the graphic-geometric modelling elaborated through the images of the architecture thus obtained communicates results that exclude simulation. In fact, between virtuality and simulation there is a distance and a difference that makes them diametrically opposed both in their content and in

the outcomes. In an explicit way, the virtual corresponds to the comparison with reality, and the simulation denies it, therefore it narrates and describes a fake.

MORPHOLOGICAL MODELLING

Introspection relating to the study of the architectural object or of figures-images of geometric solids and the profound research inherent in their nature-structure presupposes a knowledge of the systems that can investigate them, which are also made explicit in the methods and procedures of representation that throughout the history of descriptive and projective drawing have been codified and become widely used. This last assumption may not yet be sufficient. Investigating the architectural object and

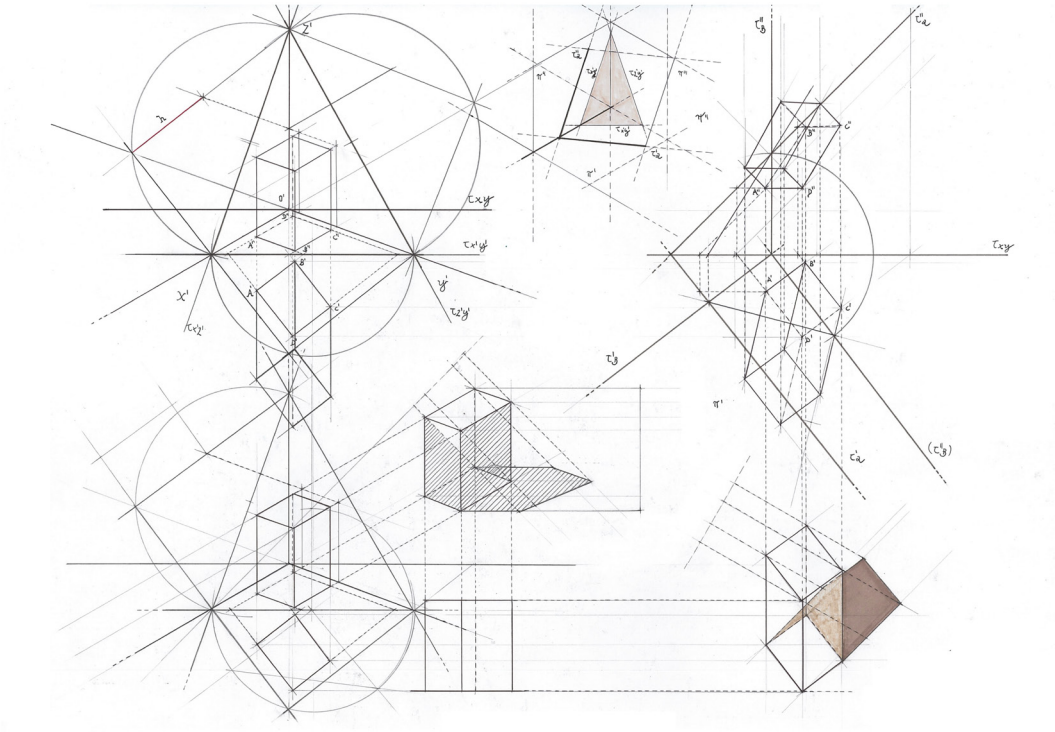


Fig. 1 Andrea Donelli, *Demonstration between separate models referred to the homological relations between the parallel projection and orthogonal projection*, 2021, china, marker, lapis, on cardboard, 29.7x49.0 cm. Author's archive.

relating it in its different aspects does not mean knowing how to describe it, draw it or having to accurately represent it. Particularly effective in relation to the graphic-geometric operations to draw and represent the object in its specific relations, is to recognize its characteristics and to grasp, graphically and geometrically, its specific homological peculiarities. This consists above all in a particular way in passing from one method of representation to another (Figure 1).

Furthermore, it is not secondary, but rather fundamental, to identify and use a research approach compatible with a form of representation. It is therefore a question of being able to produce a consequential and logical description decreed by the systems of representation and verified with descriptive and projective graphic models. In this regard, the concept of model is more than ever appropriate, since it can be considered in its hypothetical double meaning: indispensable for experiencing a phenomenon, but also a productive datum which involves a certain figurative synthesis projected onto a plane or on several planes. The first concept of the model in architecture and/or descriptive geometry is understood as the structure of the representation system: that is, it concerns the structure itself relative to the method referred to the representation method on which the object is graphically and geometrically arranged, in the way it is intended to be scientifically and technically described. The second concept relating to the model is to be understood in its graphic visualization defined by the representation of the described object and consequently obtained as a result of a visualized and concrete result. In this assumption, the modelling that describes in its complete aspects and in its unitary terms the concept of model in its double meaning explained above is particularly effective. For example, parallel projection or parallel perspective is a scientific and technical system defined by the traces that have a geometric and graphic origin from the proof inferred from the fundamental triangle or of the traces necessary to demonstrate the origin

of the orthogonal axial triple, z'-y'-x'. Therefore, a structure relative to a representation model is generated in which are arranged the same traces necessary to draw the image relative to another system considered as the figurative model which is the oblique object thus represented as axonometric (Figure 2).

It can be deduced that from a figure positioned in the first projection in true scale, an oblique three-dimensional figure is obtained, capable of describing by continuity and contiguity the model governed by the systematicity defined by the geometric projections or in which homological relations exist. In fact, homology –from the Greek *ὁμολογία*, homoios 'similar', 'equal' and logos 'discourse', that is, same logic, obviously same discourse– takes on the meaning of a precise logical correspondence between two things, therefore what happens in one happens also in the other

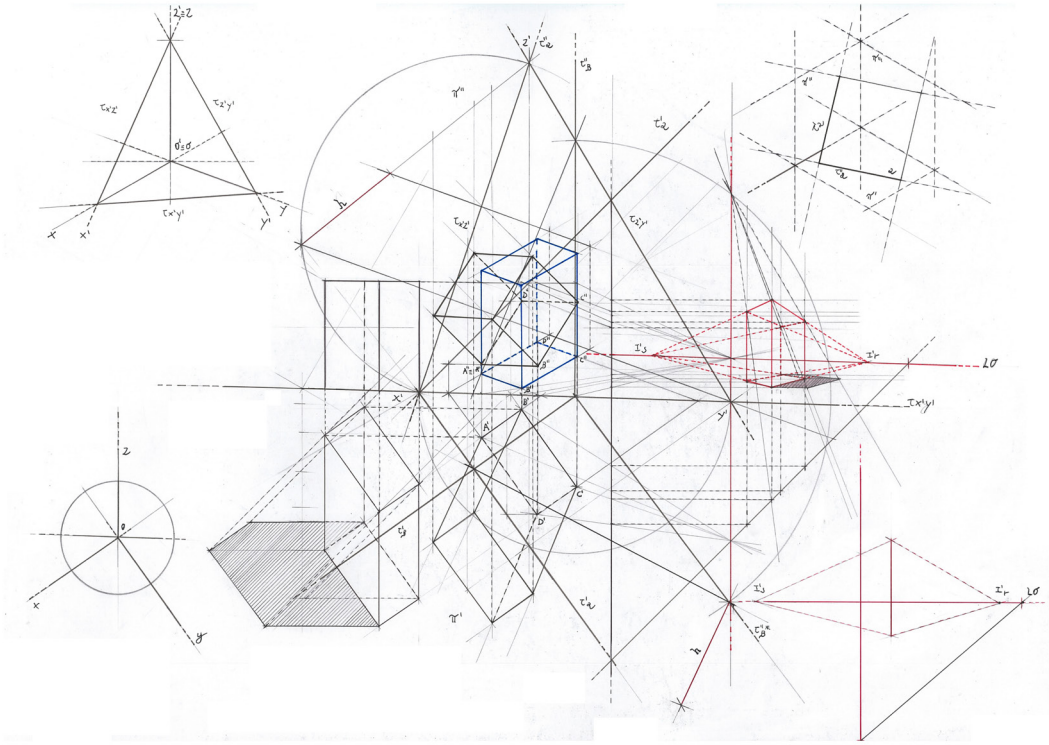


Fig. 2 Andrea Donelli, *From one system to another: homological relationships between parallel projection, orthogonal projection; perspective and axonometric images*, 2021, china, marker, lapis, on cardboard, 29.7x49.0 cm. Author's archive.

by reason of the same logic. On the basis of this affirmation, we can reiterate that this dual relationship also determines the correspondences relative to the projections between entities, fixing and establishing themselves in all systems of representation as a constant and undisputed homological relationship. These facts indicate from a theoretical and philosophical point of view a concordance and a consequence of connotations of graphic-geometric relations considered fundamental, which tend in their abstraction to become isolated facts for their own sake. The application, or better said, the concretization of these descriptive graphic speculations, finds its implementation through forms of research and investigation as already anticipated in relation to the architectural object. We are urged to investigate the reality of the object and consequently to have to represent it through drawing based on geometric and projective rules. We have seen how these rules originate, in their essence, specific and precise relationships that we cannot escape from. This holds whether they are homological, and therefore continuous and contiguous operations on planes of a geometric nature, or exclusively applicative projections and operations with points and lines. From this constant questioning, knowledge is articulated and developed. The investigation and introspection referred to the architectural object of study produces tension, creativity and curiosity that determine the desire to control the object, reveal its multiple articulations, consider its different components, generic and particular, whether they are objective as well as, subsequently, of a subjective nature. In this way, through these solicitations, we are led to consider the reality of the object. Care must be taken so that this research does not risk signifying and falling into mere subjectivism. To carry out this form of research, descriptive and projective geometry are essential to the technical-scientific drawing that allows us to undertake the specific and appropriate investigations. On this subject, Mauro Lena writes:

there must be no things that are in themselves, what is

> produced (the sense of knowledge) is not the object seen by a specific person, in a specific way, which gives a specific interpretation of the object. The object is therefore what emerges from this dual relationship, it is the reality we know, transferred in space, in representation, in judgment, in language. (1994, pp. 177, 178)

It constitutes a fact of induction in the literal meaning of 'bringing in', but also of 'calling to oneself', 'to draw to oneself', and it is a procedure that, starting from individual particular cases, tries to establish a universal law. The object analysed, investigated, and understood in this way is the result decreed by the systems of representation, by the rules and by the 'secret' correspondences or 'suspicious coincidences' as in a sort of inductive inference that gradually becomes constituted in determining the object considered or evaluated in its entirety or in its partiality. The studied object must satisfy the relationship that is established between its own form and the research and thought in which it is represented. How then can the object be described? The form of representation that best reproduces the belonging, the singularity of the facts, also the result of a model visualized in the abstract is orthogonal projection with homology. A question that appears spontaneous is: what can be used to draw and represent an object through an operation of orthogonal projection with homology? This question arises frequently, especially for those who do not know the speculative potential that these exercises constitute in their essence. The operation relating to the orthogonal projection with homology that constitutes its foundations according to Desargues' speculations is determined by the following known properties: Desargues' theorem of homological triangles states that two triangles, ABC and A'B'C', without vertices and sides in common, are prospective with respect to a point if and only if they are prospective with respect to a line. In fact, the homological triangles referring to two triangles of vertices A, B, C and A', B', C', are said to be: perspective with respect to a point if the lines generated by the pairs of

corresponding vertices <A, A'>; <B, B'>; <C, C'>, pass through a point O, which is called the centre of homology; perspective with respect to a line if the pairs of homologous sides <A, B>, <A', B'>; <A, C>, <A', C'>; <B, C>, <B', C'>; intersect at points aligned on a straight line r, which is called the homology axis. The definition inherent in the theorem of homological triangles by Desargues is thus evermore important. The final result is the obtaining of different positions of the object or figures, whose metric and non-metric values constitute an additional form of abstraction of the object itself, placing it in degrees of extreme synthesis and in virtual contexts (Figure 3).

PRINCIPLE OF VIRTUAL WORKS IN DESCRIPTIVE GEOMETRY

With the definition of the 'principle of virtual works' in descriptive geometry, we intend to make, borrowing from the philosophical theory of the science of constructions, a precise reference to the fundamental bases decreed by virtuality which, through their concepts, already set and determined the method from about the second half of the eighteenth century. For example, Desargues' homological triangles theorem constitutes a very important and fundamental datum and result as far as descriptive and projective geometry is concerned. On the basis of this result, further comparable, i.e. virtual, conjectures can be obtained in a set of axioms that govern the use of only three primitive concepts: point, line and incidence. From this we deduce that the principle of Desargues' theorem holds for every coordinatizable projective space, as well as for every projective space whose dimension is different by two and for every projective plane in which Pappus' theorem holds. There are also other so-called non-Desarguesian projective planes, for example the Moulton plane, for which there is no Desargues theorem. From these assumptions it is possible to understand a sort of process of abstraction, or rather the setting of virtual conditions, due

to the fact that in projective geometry the historical quarrel between synthetical and analytical thought reappears in relation to the question of the distinction between geometric properties and those metrics. In fact, taking as an example an exercise relating to the orthogonal projection with homology, the metric value of the image is not recognized after having projected it as a figure on a generic, inclined plane, and subsequently after having carried out the operation of overturning this same generic plane on a horizontal plane of the dihedral. In conclusion, the projected image does not have metric properties, may be proportionate –true form– for effects of relations –harmonic whole–, but does not acquire properties in true magnitude. This apparently obvious result, on the other hand, opens up an important question regarding thought about the concrete operations to carry out with orthogonal projection con homology and, consequently, on its

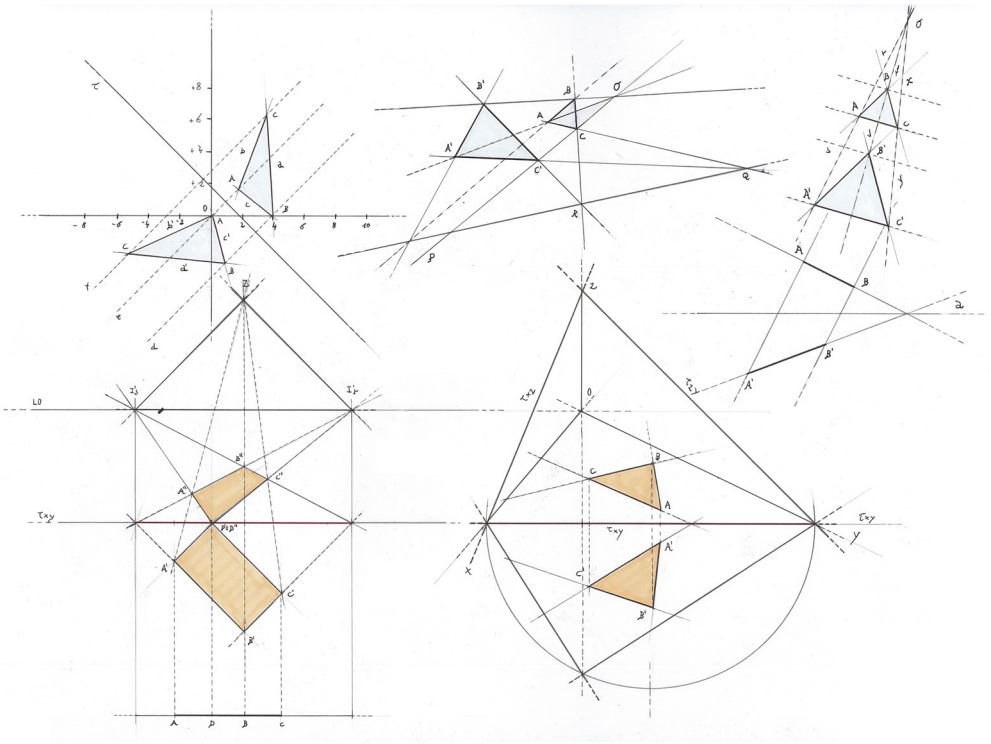


Fig. 3 Andrea Donelli, *Homological triangles*, 2021, china, marker, lapis, on cardboard, 29.7x49.0 cm. Author's archive.

current valid utility and necessity which is often questioned in relation to the means and instruments in place. If historically, on the one hand, the fundamental concept that was used in the synthetic presentation of projective geometry, that is the basic concept defined by the cross-ratio decreed by the four points on a straight line, was introduced using the lengths of the intervals, on the other hand the introduction of projective coordinates was also based on the distances from the sides of the reference triangle or, alternatively, on other concepts of metric structure. In this case the problem arose of rethinking projective geometry without making use of metric concepts. This question attracted the attention of von Staudt, who, like Möbius, a student of Gauss, devoted himself to the attempt to formulate a projective geometry independent of any metric concept. To realize and solve this problem von Staudt started from the observation that if a correspondence between forms of the first kind preserves the properties of harmonic ensembles, then it also preserves the properties of cross ratios. Since harmonic sets are thus definable with exclusively graphic and not specifically analytic constructions, his concept of the definition of projectivity does not use metric concepts. If this aspect resolves the question of the past due to the contrasts between the thinking positions of synthetics with respect to analytics, it still does not fully resolve the usefulness and need to represent an object with the use of homology. The title of this contribution intends to investigate the object that has fallen into its reality, but not an object as it seems to have to be perceived in a purely obvious vision, or, as has already been said, seen by a certain person, in a certain way. The investigated object, while remaining faithful to its reality, needs to be broken down, and therefore also reassembled or compared through virtual passages (Figures 4, 5).

In this way, the research on the whole and on the part in architecture takes on meaning and significance, demonstrated through morphological modelling processes. The object examined in this way is freed from metric issues, while also

making use of them and benefiting from them when they are deemed indispensable to acquire and consider for the object itself in order to obtain metric values and consequently in true magnitude. The use of orthogonal projection with homology allows us to investigate and penetrate the object in order to take away a specific understanding since its graphic-geometric determination establishes the object in an abstract form but not, for this reason, unreal or as a surrogate for possible and attributable simulative definitions that do not correspond to the concept of virtuality. In this case, the graphic properties of the cross-ratio are confirmed, and a further form of research and non-metric thinking is introduced to which Möbius devoted much attention and in-depth analysis referring to the question of topological geometry. The foundation on which topology is based is defined by the two-way relations. These properties also belong to homology, as they are foundations

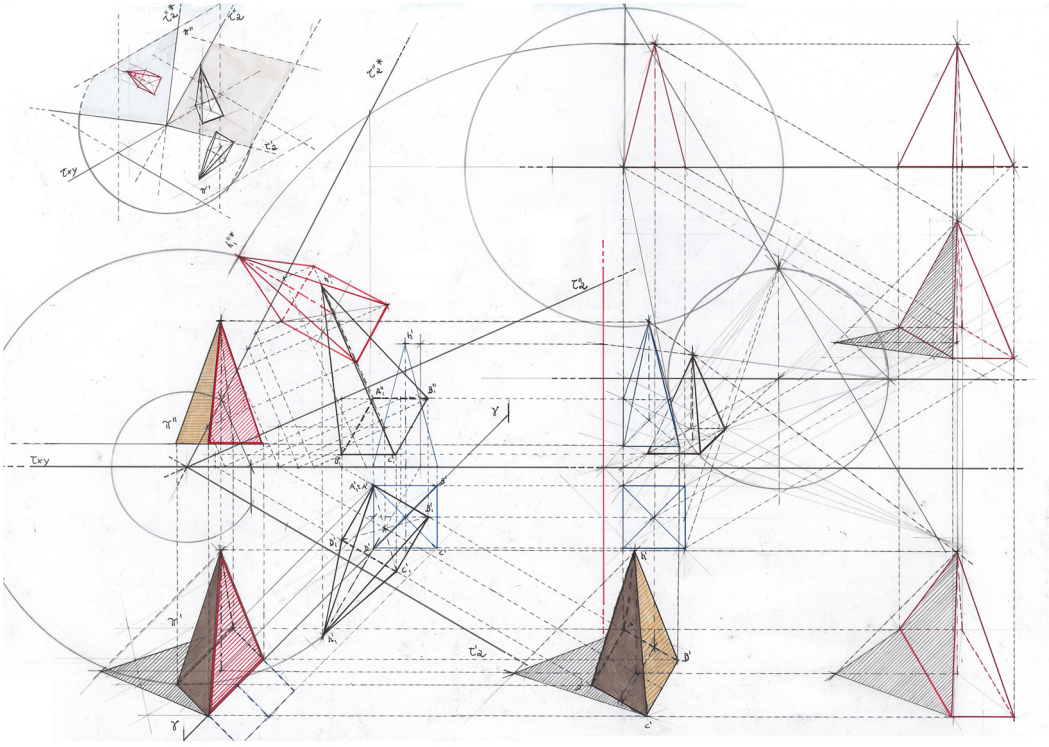


Fig. 4 Andrea Donelli, *Morphological modeling inherent to homological and co-homological relations in the representation of solid figures*, 2021, china, marker, lapis, on cardboard, 29.7x49.0 cm. Author's archive.

and connotations that unite all descriptive, projective and even topological geometric thought. This research is interested in starting a process of investigation, here exposed in its assumptions and in its declarations of intent, to proceed to consider the reality of the object starting from its metric peculiarities but to arrive at a form of abstraction supported by virtual issues and not necessarily metrics. Virtuality is a support of the process that allows us to investigate reality, specifically, to control and reason around the understanding of the object, synthesizing a method of knowledge and therefore of representation. The concept of virtuality should not be confused with that of simulation. It would be a mistake to consider the virtual process to be associated with formalistic and baseless and easily alterable outcomes. In fact, for Jean Baudrillard, simulation is pretending to have what one does not have.

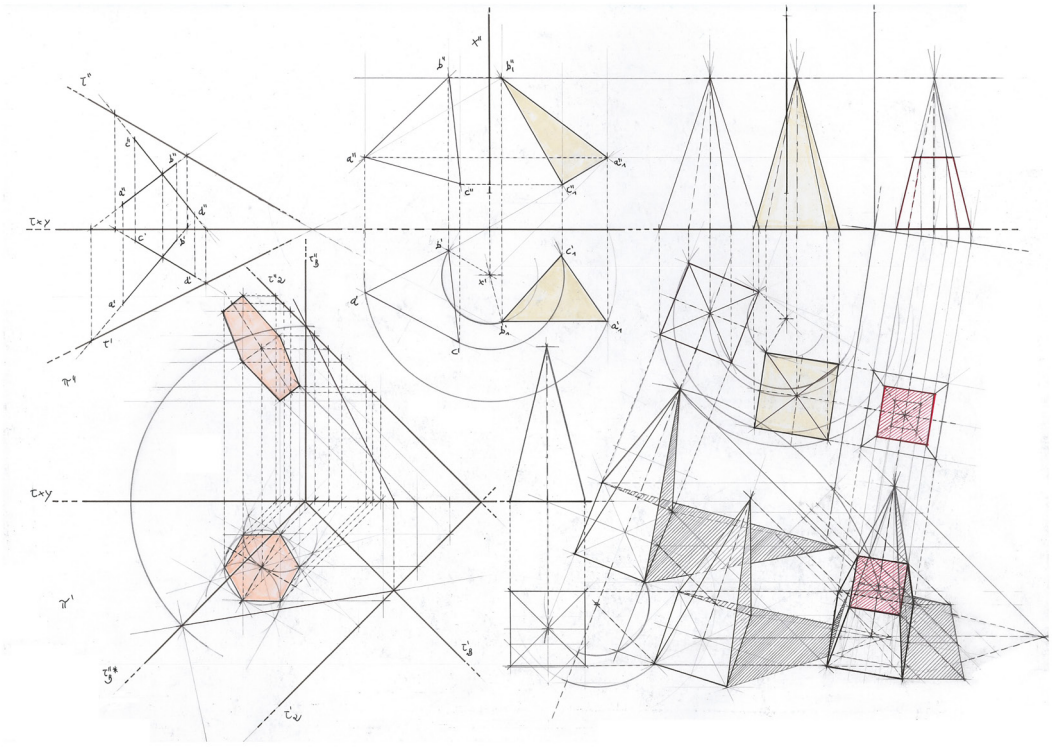


Fig. 5 Andrea Donelli, *Morphological modeling: homological relationships between orthogonal projection and axonometry*, 2021, china, marker, lapis, on cardboard, 29.7x49.0 cm. Author's archive.

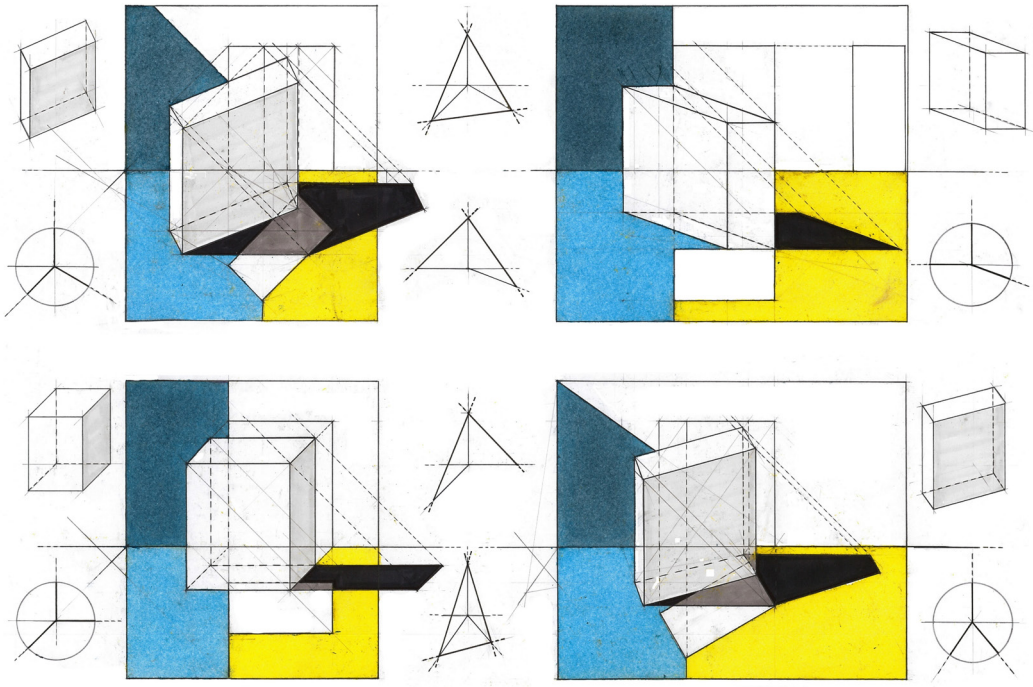
> Those who pretend to be ill can simply go to bed and make others believe they are ill. Those who simulate a disease produce some of its symptoms in themselves. So, pretending is only masking; simulation, on the other hand, undermines the difference between 'true' and 'false' and between 'real' and 'imaginary' (Baudrillard, 1994, pp. 2 -5).

CONCLUSIONS

The concept of homology belongs to all of the graphic-geometric relationships. In fact, "to represent scientifically signifies to acquire and transmit the knowledge of the form of a real or imaginary entity and of the rules that underlie it" (Catalano, 1994, p. 29). Orthogonal projection with homology, like parallel projection, belongs to the category of metric and non-metric representation. Representation in space of an object that is drawn and observed through non-metric applications allows us to also think of abstract forms or images, in this way isolated from a predictable form of subjective judgement. In fact, the analysis of the object, and the deep investigation of its nature-structure, must result in such a way as to be controlled by the forms of representation, whether they are first metric and subsequently non-metric, which involve a particular gnoseological constitution (Figure 6). The object represented through the structure of a graphic-geometric system in non-metric form as an aid to the orthogonal projection and with the comparison with the homology of a regular solid projected on an inclined generic plane, contributes through this experience to represent a very special and unique synthesis. In this way there is a clear demonstration of the uniqueness and unrepeatability of the shape and position of the image, which are defined by the projective geometric properties in space. The relationships or the models that are established between the objects projected on the geometric planes constitute a comparison determined by a principle of virtual works, i.e. by relationships that relate

to each other through the dual exchange due to graphic-geometric modelling, with the important contribution of being able to switch from one representation to another. These transfers generate relationships, virtual models with the reality of the object and with the examination of the images that are returned through the different positions they assume in the geometric planes.

Fig. 6 Andrea Donelli, *Morphological modeling referred to the application of axonometry relating to the different positions of the object between the real and virtual image in contiguity with the projection values of the shadow measurement*, 2021, china, marker, lapis, on cardboard, 29.7x49.0 cm. Author's archive.



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